Controller Realizations of a Teleoperated Dual-Wrist Assembly System with Finite Word Length Considerations

Robert S. H. Istepanian, Jun Wu, and James F. Whidborne

Abstract—This paper presents some implementation issues of finite-precision optimal and quasioptimal sparse H∞ controller structures applied to a teleoperated dual-wrist assembly system. The relevant stability robustness measures and the optimization search procedures for both the optimal and the quasioptimal sparse Finite Word Length (FWL) controller realizations are described. Comparative analytical and numerical results show that the sparse FWL controller realizations provide minimum bit lengths, with maximum trivial elements in the controller coefficients and higher stability bounds compared to nonoptimal realization. The results illustrate that the FWL methodology presented provides better implementation structures, increased computational speed and FWL stability for teleoperated motion-scaling control systems.

Index Terms—Controller realization, digital control, finite word length effects, microsurgery, simulated annealing, sparse realization, stability, teleoperation control.

I. INTRODUCTION

TELEOPERATION may be viewed as a branch of robotics concerned with the manipulation in environments or spaces generally inaccessible to man. The basic teleoperation system consists of a slave device tracking a master device directly manipulated by a human operator. When the master is also actuated based on sensor signals from the slave, such a system is said to be bilateral. The kinesthetic sensations provided in bilateral teleoperators can substantially enhance operator performance both in speed and safety [1].

The bilateral teleoperation system studied in this paper is a dual-wrist assembly developed in the University of British Columbia. This dual-wrist assembly is a prototype telerobotic system designed for use in microsurgery experiments [2]. An \( H_\infty \) digital controller is used in this dual-wrist assembly. Hence the dual-wrist assembly control system is in fact a sampled data system \( S_1 \) shown in Fig. 1.

The sampled data system consists of the continuous-time model of the dual-wrist assembly \( P(s) \), the digital controller \( C(z) \), the sampler (A/D converter) \( S_h \) and the hold (D/A converter) \( H_h \) with sampling period \( h \). It is known from sampled data system theory [3] that generically the stability of the dual-wrist assembly control system \( S_1 \) is equal to the stability of discrete time system \( S_2 \) shown in Fig. 2, where \( P(z) = S_h P(s) H_h \) is the discretized \( P(s) \). Let \( (A_z, B_z, C_z, 0) \) be a state-space description of \( P(z) \) (\( P(z) \) is assumed to be strictly causal), \( A_z \in \mathbb{R}^{m \times m} \), \( B_z \in \mathbb{R}^{m \times d} \), \( C_z \in \mathbb{R}^{p \times m} \). Let \( (A_C, B_C, C_C, D_C) \) be a state-space description of \( C(z) \), \( A_C \in \mathbb{R}^{m \times n}, B_C \in \mathbb{R}^{m \times q}, C_C \in \mathbb{R}^{p \times n}, D_C \in \mathbb{R}^{p \times q} \). Then the stability of the dual-wrist assembly control system \( S_1 \) is dependent on the poles of the closed-loop system matrix

\[
\overline{A} = \begin{bmatrix}
A_z + B_z D_C C_z & B_z C_C \\
B_C C_z & A_C
\end{bmatrix}.
\]

The dual-wrist assembly is a rather complex system with six degree-of-freedom (DOF) motions. The control objectives of such a system are to guarantee closed-loop stability and make it as close as possible to an “ideal teleoperator.” Detailed control design objectives and a general description of the teleoperation system and the relevant \( H_\infty \) controller design framework are given in [2]. However, it is well known that such applications require high sampling rates in order to provide the required computational speed and minimum delay in the master/slave control system structure. Hence, fixed-point controller implementations are preferred in such critical applications because they offer high...
speed processing as well as the advantages of low memory space, low cost and simplicity. However, significant finite-word-length (FWL) effects are inherent in such structures and so FWL stability issues [5], [10], [11] must be addressed. This paper considers the problem of obtaining the optimal and the quasi-optimal sparse FWL controller structures for a teleoperated dual-wrist assembly discrete-time $\mathcal{H}_\infty$ controller. Comparative numerical results of the optimal and sparse quasi-optimal FWL controller realizations are presented with the relevant frequency responses of the system. These results provide an applied design framework for the FWL implementation issues of enhanced teleoperated optimal controller structures with improved stability, minimum memory requirements and higher computational speed for such critical teleoperated robotic systems.

II. A FWL Stability Robustness Measure

Any linear system which can be modeled as a transfer function matrix can also have an infinite number of state-space descriptions. In fact, if $(A_0, B_0, C_0, D_0)$ is a state-space description of a digital controller, $C(z)$, of the dual-wrist assembly, all state-space descriptions of $C(z)$ form the set

$$ S_C := \{(A_C, B_C, C_C, D_C) | A_C = T^{-1} A 0 T, B_C = T^{-1} B 0 C C_C = C 0 T, D_C = D 0 C\} \quad (2) $$

where $T$ is any nonsingular matrix, called a similarity transformation. In this paper, a $(A_C, B_C, C_C, D_C)$ in $S_C$ is called a realization of $C(z)$. Denote $N = (I + n)(q + n)$ and

$$ X \triangleq \begin{bmatrix} D_C & C_C \\ B_C & A_C \end{bmatrix} = \begin{bmatrix} p_1 & p_{q+1} & \cdots & p_{N-1+n} \\ p_2 & p_{q+2} & \cdots & p_{N-2+n} \\ \vdots & \vdots & \ddots & \vdots \\ p_1+n & p_{q+1}+n & \cdots & p_N \end{bmatrix} \quad (3) $$

We will also refer to $X$ as a realization of $C(z)$. From (1), we know

$$ \hat{A}(X) = \begin{bmatrix} A_z & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} B_z & 0 \\ 0 & I \end{bmatrix} X \begin{bmatrix} C_z & 0 \\ 0 & I \end{bmatrix} \triangleq M_0 + M_1 X M_2. \quad (4) $$

In practice, due to the FWL effect, $X$ is perturbed to $X + \Delta X$, where

$$ \Delta X \triangleq \begin{bmatrix} \Delta p_1 & \Delta p_{q+1} & \cdots & \Delta p_{N-1+n} \\ \Delta p_2 & \Delta p_{q+2} & \cdots & \Delta p_{N-2+n} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta p_1+n & \Delta p_{q+1+n} & \cdots & \Delta p_N \end{bmatrix} \quad (6) $$

and each element of $\Delta X$ is bounded by $\varepsilon/2$, i.e.,

$$ \mu(\Delta X) \triangleq \max_{i \in \{1, \ldots, N\}} \left| \frac{\partial \lambda_i}{\partial p_i} \right| \leq \frac{\varepsilon}{2} \quad (7) $$

where $\mu$ represents the maximum possible perturbation on a controller parameter. For a fixed-point processor of $B_X$ bits

$$ \varepsilon \triangleq 2^{-(B_x - B_X)} \quad (8) $$

where $2^{B_X}$ is the biggest normalization factor such that each element of $2^{-(B_x - B_X)} X$ is absolutely not bigger than 1. With the perturbation $\Delta X$, a closed-loop pole $\lambda_i(\hat{A}(X))$ is moved to $\lambda_i(\hat{A}(X + \Delta X))$ which may be outside the open unit disk and hence be unstable.

Next we will derive an FWL stability robustness measure to describe the FWL stability performance of a controller realization $X$ of the dual-wrist assembly. First of all, when the FWL error $\Delta X$ is small we have [5]

$$ \Delta \lambda_i \triangleq \lambda_i(\hat{A}(X + \Delta X)) - \lambda_i(\hat{A}(X)) \quad (9) $$

$$ \approx \sum_{j=1}^{N} \frac{\partial \lambda_i}{\partial p_j} \Delta p_j, \quad \forall \ i \in \{1, \ldots, m+n\}. \quad (10) $$

It follows from the inequality (derived easily from the Cauchy inequality)

$$ \left( \sum_{j=1}^{N} a_j \right)^2 \leq \sum_{j=1}^{N} a_j^2 \quad (11) $$

that for all $i \in \{1, \ldots, N\}$

$$ |\Delta \lambda_i| \leq \sqrt{N} \left( \sum_{j=1}^{N} \left| \frac{\partial \lambda_i}{\partial p_j} \right|^2 \right) \leq \mu(\Delta X) \sqrt{N} \sqrt{\sum_{j=1}^{N} \left( \frac{\partial \lambda_i}{\partial p_j} \right)^2}. \quad (12) $$

Define

$$ \mu_{\Delta}(X) \triangleq \min \left\{ \frac{1 - |\lambda_i(\hat{A}(X))|}{\sqrt{\sum_{j=1}^{N} \left( \frac{\partial \lambda_i}{\partial p_j} \right)^2}} \right\} \quad (13) $$

If $\mu(\Delta X) < \mu_{\Delta}(X)$, it follows directly from (12) and (13) that

$$ |\Delta \lambda_i| < 1 - |\lambda_i(\hat{A}(X))| \quad (14) $$

which means $S_1$ remain stable under the FWL errors $\Delta X$. Thus we have the following proposition.

**Proposition 1:** Dual-wrist assembly control system $S_1$ is FWL stable when

$$ \mu(\Delta X) < \mu_{\Delta}(X). \quad (15) $$

For a given dual-wrist assembly controller realization $X$, Proposition 1 shows that the dual-wrist assembly can tolerate those FWL perturbations $\Delta X$ in which each element is less than $\mu_{\Delta}(X)$. The bigger $\mu_{\Delta}(X)$ is, the larger the FWL errors that the system can tolerate. Thus $\mu_{\Delta}(X)$ is stability robustness measure describing the FWL stability of a controller realization $X$ of the dual-wrist assembly. More importantly, let

$$ \hat{B}_{\Delta X} \triangleq \text{Im}[- \log_2(\mu_{\Delta}(X))] - 1 + B_X \quad (16) $$

where $\text{Im}[x]$ rounds $x$ up to the nearest integer and $\text{Im}[x] \geq x$. From (7), (8) and Proposition 1, we know that the dual-wrist assembly control system $S_1$ is still stable when the digital controller realization $X$ is implemented with a fixed-point processor of $\hat{B}_{\Delta X}$ bits. Knowing that $\mu(\Delta X) < \mu_{\Delta}(X)$ is a sufficient condition, we can see $\hat{B}_{\Delta X}$ represents the upper estimate of $B_{\Delta X}$. $B_{\Delta X}$ is the smallest word length that, when used to implement a fixed-point structure $X$, can guarantee the closed-loop stability of the dual-wrist assembly control system.
Theorem 1: Suppose \( \overline{A}(X) = M_0 + M_1 X M_2 \) in (5) is diagonalizable, and \( \lambda_i \) as its eigenvalues. Let \( x_i \) be a right eigenvector of \( A \) corresponding to the eigenvalue \( \lambda_i \). Denote \( M_x \triangleq [x_1 \ x_2 \ \cdots \ x_{m+n}] \) and \( M_y \triangleq [y_1 \ y_2 \ \cdots \ y_{m+n}] \) where \( H \) denotes the transpose and conjugate operation and \( y_i \) is called the reciprocal left eigenvector corresponding to \( \lambda_i \). Then

\[
\frac{\partial \lambda_i}{\partial X} = \begin{bmatrix}
\frac{\partial \lambda_i}{\partial q_{i_1}} & \cdots & \frac{\partial \lambda_i}{\partial q_{i_{m+n-1}}}
\end{bmatrix} = M_y^H x_i^HM_y^2 (17)
\]

where the superscript * denotes the conjugate operation.

Proof: Let \( \alpha \) be a variable independent of \( M_0, M_1 \) and \( M_2 \). It follows from \( y_i^H x_i = 1 \) that

\[
\frac{\partial y_i^H}{\partial \alpha} x_i + y_i^H \frac{\partial x_i}{\partial \alpha} = 0. \tag{18}
\]

Noting \( A x_i = \lambda_i x_i \), one steadily has \( \lambda_i = y_i^H \overline{A} x_i \) and hence

\[
\frac{\partial \lambda_i}{\partial \alpha} = \left( \frac{\partial y_i^H}{\partial \alpha} \lambda_i x_i + y_i^H \frac{\partial \overline{A}}{\partial \alpha} x_i \right) + y_i^H \frac{\partial \overline{A}}{\partial \alpha} x_i \tag{19}
\]

It follows from (18) and \( y_i^H \overline{A} = \lambda_i y_i^H \) that

\[
\frac{\partial \lambda_i}{\partial \alpha} = \left( y_i^H M_1 \right) (k)(M_2 x_i)(j) \tag{20}
\]

Let \( \alpha = \beta_{kj} \), the \((k, j)\)th element of \( \alpha \), one has

\[
\frac{\partial \lambda_i}{\partial \alpha} = \left( y_i^H M_1 \right)(k)(M_2 x_i)(j) \tag{21}
\]

where \( (y_i^H M_1)(k) \) and \( (M_2 x_i)(j) \) are the \( k \)th and \( j \)th element of \( y_i^H M_1 \) and \( M_2 x_i \), respectively. This leads to (17).

Based on (13) and Theorem 1, a MATLAB program to compute the FWL stability robustness measure \( \mu_1(X) \) has been developed by the authors which provides direct FWL controller realizations based on the \( H_\infty \) control results given in [2]. In this dual-wrist assembly structure, \( m = 4, n = 10, l = 2, q = 2 \), hence the control system has 14 closed-loop poles in total and its \( H_\infty \) controller realization \( X \) has 144 elements affected by the FWL coefficient errors. Based on the stability method described earlier, we computed \( \mu_1 \) for \( X_{\text{min}} \) and \( \mu_1 \) for \( X_{\text{max}} \). The absolute largest element of \( X_{\text{min}} \) is given as 4.2341 \times 10^4 which means that \( B_X \) of \( X_{\text{min}} \) is at least 16-bits. Then from (16) we obtained the relevant \( D_{\text{min}} = 20 \). Therefore we conclude that for the control system of the dual-wrist assembly system given in [2], up to 29 bits may be required to keep the system stable when the fixed-point \( X_{\text{min}} \) realization of the \( H_\infty \) controller realization is used.

III. OPTIMAL CONTROLLER REALIZATION OF THE DUAL-WRIST ASSEMBLY WITH FIXED-POINT IMPLEMENTATION

Since different realizations \( X \) have different right eigenvectors and left eigenvectors, we know from Theorem 1 that different controller realizations \( X \) of the dual-wrist assembly yield different \( \mu_1(X) \). The realizations that yield a maximum \( \mu_1 \) lead to the smallest \( D_{\text{min}} \), which means that the dual-wrist assembly \( H_\infty \) digital controller implemented with finite-precision must satisfy the minimum hardware requirements in terms of minimum word length (i.e., optimized controller data path hardware design) with maximum stability. Therefore, the optimal control realization problem is given as

\[
v = \max_{\lambda \in \lambda \in H_\infty} \mu_1(X). \tag{23}
\]

Denote

\[
X_0 = \begin{bmatrix}
D_C & 0 \\
B_C & C_C
\end{bmatrix} \tag{24}
\]

Then

\[
X = X(T) = \begin{bmatrix}
I & 0 \\
0 & T^{-1}
\end{bmatrix} X_0 \begin{bmatrix}
I & 0 \\
0 & T
\end{bmatrix} \tag{25}
\]

and

\[
\overline{A}(X) = \begin{bmatrix}
I & 0 \\
0 & T^{-1}
\end{bmatrix} \overline{A}(X_0) \begin{bmatrix}
I & 0 \\
0 & T
\end{bmatrix}. \tag{26}
\]

Obviously, \( \lambda^0 \triangleq \lambda_i(\overline{A}(X_0)) = \lambda_i(\overline{A}(X)) \). From (26), applying Theorem 1, we have

\[
\frac{\partial \lambda_i}{\partial X} = \begin{bmatrix}
0 & 0 \\
I & 0
\end{bmatrix} \frac{\partial \lambda_i}{\partial A} X_0 \begin{bmatrix}
I & 0 \\
0 & T^{-1}
\end{bmatrix}. \tag{27}
\]

Hence, the problem given by (23) is equivalent to

\[
v = \max_{\lambda \in \lambda \in H_\infty} \min_{T} \left\{ \frac{1 - |\lambda^0|}{\sqrt{N}} \right\} \times \left\{ \begin{bmatrix}
I & 0 \\
0 & T^{-1}
\end{bmatrix} \frac{\partial \lambda_i}{\partial A} X_0 \begin{bmatrix}
I & 0 \\
0 & T^{-1}
\end{bmatrix} \right\} F \tag{28}
\]

where \( \| \cdot \|_F \) denotes the Frobenius norm. This gives

\[
v = \max_{\lambda \in \lambda \in H_\infty} \min_{T} \left\{ \frac{1}{\sqrt{N}} \right\} \times \left\{ \begin{bmatrix}
I & 0 \\
0 & T^{-1}
\end{bmatrix} \Phi_i \begin{bmatrix}
I & 0 \\
0 & T^{-1}
\end{bmatrix} \right\} F \tag{29}
\]

where

\[
\Phi_i \triangleq \frac{\partial \lambda_i}{\partial X_0} X_0 \frac{1}{1 - |\lambda^0|} \tag{30}
\]

are fixed matrices which are independent of \( T \). Thus we can give the following expression for the cost function

\[
f(T) = \min_{i \in \{1, \ldots, m+n\}} \frac{1}{\sqrt{N}} \times \left\{ \begin{bmatrix}
I & 0 \\
0 & T^{-1}
\end{bmatrix} \Phi_i \begin{bmatrix}
I & 0 \\
0 & T^{-1}
\end{bmatrix} \right\} F \tag{31}
\]

and lead to an unconstrained optimization problem

\[
v = \max_{T} f(T). \tag{32}
\]
Because \( f(T) \) is a nonsmooth and nonconvex function, the optimization procedure must be based on a direct search without the aid of cost function derivatives. Conventional optimization methods for this type of problem, such as the simplex algorithm [6], generally can only find a local minimum. It is therefore important to use an efficient and preferably global optimization procedure. We adopt a global optimization strategy based on simulated annealing to search for a true global optimal controller realization \( X_{\text{opt}} \) of the dual-wrist assembly. Details of the strategy are beyond the scope of this paper and are discussed in detail in [7], [12].

Based on (29) and the simulated annealing optimization algorithm, a MATLAB program for optimizing \( \mu_2(X) \) has been developed by the authors to find the relevant optimal FWL controller structures of the teleoperated system. The optimal controller realization \( X_{\text{opt}} \), the relevant optimal similarity transformation \( T_{\text{opt}} \) such that \( X_{\text{opt}} = X(T_{\text{opt}}) \) and \( v = \mu_1(X_{\text{opt}}) = 1.5884 \times 10^{-3} \) are obtained. The absolute biggest element of \( X_{\text{opt}} \) in this case is \( 3.3771 \times 10^4 \) which means that \( B_X \) of \( X_{\text{opt}} \) is 16 bits and hence the relevant bit-length \( B_{\text{min}}^{\text{opt}} = 25 \).

### IV. Sparse FWL Realizations of the Dual-Wrist Assembly with FWL Considerations

Parameters in a controller realization such as 0, +1, and \(-1\) (called trivial parameters) not only have no rounding error, but do not require any multiplication operations and hence also provide a more efficient implementation. Hence, in this section, we derive the quasioptimal FWL controller structure that provides the maximum number of trivial elements in the \( \mathcal{H}_\infty \) controller structure while maintaining an improved stability measure compared to the nonoptimal realizations described earlier.

First of all, by modifying the measure introduced in the last section, another FWL robust stability measure is proposed as

\[
\mu_2(X) = \min_{i \in [1, \ldots, m+n]} \left[ \frac{1 - |\lambda_i(A(X))|}{N_\delta \sum_{j=1}^{n} \delta(p_j) \frac{\partial \xi}{\partial T}} \right]^{1/2}
\]

where \( N_\delta \) is the number of the nontrivial elements in \( X \) and the function

\[
\delta(p) = \begin{cases} 0 & \text{if } p = 0, +1, -1 \\ 1 & \text{otherwise} \end{cases}
\]

since there can be no rounding on the trivial parameters \( 0, +1, -1 \). Obviously, \( \mu_2(X) \) is more practical than \( \mu_1(X) \), and

\[
B_{\text{min}}^{\text{opt}} = \text{Int}[\log_2(\mu_2(X))] - 1 + B_X
\]

is a better upper estimate of \( B_{\text{opt}}^{\text{min}} \).

For the dual-wrist assembly control system, it is especially desirable for real-time implementation to use realizations that not only yield good performance \( \mu_2(X) \) but also yield as many trivial parameters as possible. Some canonical controller forms are sparse, but their corresponding \( \mu_2(X) \) may be poor and hence such realizations are usually unsuitable. Here, we use an algorithm developed in [8] to obtain a desirable sparse controller realization for the dual-wrist assembly controller. The details of the algorithm are described in [8] and [9]. The basic concepts and implementation steps are presented here for completeness.

The relevant steps of this FWL search algorithm are summarized in the following steps.

**Step 1)** \( \tau \) is set to be a very small positive real number, e.g., \( 10^{-5} \). The transformation matrix \( T \in \mathbb{R}^{m \times n} \) is set initially to be \( T_{\text{opt}} \) such that \( X_{\text{opt}} = X(T_{\text{opt}}) \) in calculated in Section III.

**Step 2)** Find out the nontrivial element \( \xi \) which is the nearest to zero, \( +1 \text{ or } -1 \) in \( X(T) \). Find out all trivial elements \( \{\eta_1, \ldots, \eta_r\} \) in \( X(T) \) (Suppose \( X(T) \) has \( r \) trivial elements).

**Step 3)** Choose \( S \in \mathbb{R}^{m \times n} \) such that

i) \( \mu_2(X(T + \tau S)) \) is close to \( \mu_2(X(T)) \).

ii) \( \{\eta_1, \ldots, \eta_r\} \) in \( X(T) \) remain unchanged in \( X(T + \tau S) \).

iii) \( \xi \) in \( X(T) \) is changed as nearer as possible to zero, \( +1 \text{ or } -1 \) in \( X(T + \tau S) \).

iv) \( ||S||_F = 1 \).

If \( S \) does not exist, \( T_{\text{opt}} = T \) and terminate the algorithm.

**Step 4)** \( T = T + \tau S \). If \( \xi \) in \( X(T) \) is still nontrivial, go to Step 3). If \( \xi \) in \( X(T) \) is trivial (\( \xi \) is considered as a trivial parameter when its distance from zero, \( +1 \text{ or } -1 \) is less than \( 10^{-5} \)), go to Step 2).

The key step in the algorithm described above is step 3 which guarantees that \( X(T_{\text{opt}}) \) has improved performance \( \mu_2 \) with as many trivial parameters as possible. To obtain \( S \), we need to denote

\[
V_T \triangleq \text{Vec}(T) \in \mathbb{R}^{2^2}
\]

\[
\text{Vec}(T) \text{ is the vector composed of the columns of } T \text{ taken in order. Similarly, we also denote}
\]

\[
V_S \triangleq \text{Vec}(S) \in \mathbb{R}^{2^2}
\]

With a very small \( \tau \), condition i) means

\[
\frac{\partial \mu_2}{\partial T} V_S = 0
\]

Condition ii) means

\[
\begin{cases} \frac{\partial \mu_2}{\partial T} V_S = 0 \\ \vdots \\ \frac{\partial \mu_2}{\partial T} V_S = 0. \end{cases}
\]

Denote the matrix

\[
E \triangleq \begin{bmatrix} \frac{\partial \mu_2}{\partial T} \\ \vdots \\ \frac{\partial \mu_2}{\partial T} \end{bmatrix} \in \mathbb{R}^{(r+1) \times n^2}.
\]

Hence \( V_S \) must belong to the null space \( \mathcal{N}(E) \) of \( E \). If \( \mathcal{N}(E) \) is empty, \( V_S \) does not exist and the algorithm is terminated. If \( \mathcal{N}(E) \) is not empty, it must have basis \( \{e_1, \ldots, e_t\} \) (Assume the dimension of \( \mathcal{N}(E) \) is \( t \)). Considering condition iii) requires changing \( \xi \) to its desired value as fast as possible, we should choose \( V_S \) as the orthogonal projection of \( \partial \xi / \partial T \) onto \( \mathcal{N}(E) \).
COMPARATIVE RESULTS OF DIFFERENT CONTROLLER REALIZATIONS

<table>
<thead>
<tr>
<th>Realization</th>
<th>$\mu_1$</th>
<th>$B_{\min}^{\mu_1}$</th>
<th>$\mu_2$</th>
<th>$B_{\min}^{\mu_2}$</th>
<th>$N_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{ini}$</td>
<td>$1.1734 \times 10^{-4}$</td>
<td>29</td>
<td>$1.1734 \times 10^{-4}$</td>
<td>29</td>
<td>144</td>
</tr>
<tr>
<td>$X_{opt}$</td>
<td>$1.5844 \times 10^{-4}$</td>
<td>25</td>
<td>$1.5844 \times 10^{-4}$</td>
<td>25</td>
<td>144</td>
</tr>
<tr>
<td>$X_{scp}$</td>
<td>$4.3325 \times 10^{-4}$</td>
<td>27</td>
<td>$1.1171 \times 10^{-3}$</td>
<td>25</td>
<td>63</td>
</tr>
</tbody>
</table>

Fig. 3. Comparative frequency response plots for different realizations.

Noting condition iv), we can compute $V_S$ as follows:

$$a_i = c_i \frac{\partial c_i}{\partial T} \in \mathbb{R}, \quad \forall i \in \{1, \ldots, f\} \quad (41)$$

$$w = \sum_{i=1}^{f} a_i c_i \in \mathbb{R}^{n^2} \quad (42)$$

$$V_S = \pm \frac{w}{\sqrt{w^T w}} \in \mathbb{R}^{n^2}. \quad (43)$$

With regards to the sign in 43, if $\xi$ is larger than the nearest desirable value, then the minus sign is taken, otherwise the plus sign is used.

A MATLAB program based on the above algorithm has been developed and applied to the dual-wrist assembly $H_\infty$ controller example. The resultant sparse controller realization $X_{scp}$ has 81 trivial elements and the similarity transformation $T_{scp}$ such that $X_{scp} = X(T_{scp})$ is obtained.

Table I summarizes the implementation results of the FWL realizations $X_{ini}, X_{gw},$ and $X_{scp}$ for the $H_\infty$ controller structure of the teleoperated control system and their corresponding stability measures and resultant computational elements of the controller structures. In Table I, $X_{ini}, X_{gw},$ and $X_{scp}$ represent the initial $H_\infty$ FWL controller structure, the optimal FWL control structure and the sparse controller realization, respectively. The corresponding stability measures for each estimated minimum bit-lengths derived in Sections III and IV ($\mu_1, B_{\min}^{\mu_1}, \mu_2, B_{\min}^{\mu_2}$) are given by the variables $\mu_1, B_{\min}^{\mu_1}, \mu_2, B_{\min}^{\mu_2}$, respectively. It is clear from the results shown in Table I that both the $X_{gw}$ and $X_{scp}$ realizations require fewer bits to guarantee closed-loop stability compared to the corresponding initial realization. It is also clear that the sparse realization $X_{scp}$ contains just 63 nontrivial elements ($N_x$) compared to 144 elements for the initial and optimal realizations.

Fig. 3 shows the comparative closed-loop performance of the force tracking error taken from the active operator hand force of the teleoperated dual-wrist control system with a 30-bit length processor. It is clear from these results that the performances of both $X_{gw}$ and $X_{scp}$ realizations are roughly comparable to the ideal controller performance implemented with infinite bit-lengths, while there is a clear performance difference between $X_{ini}$ and the ideal controller. The reduced numerical and computational load of the $X_{scp}$ realization in real-time environments is also advantageous in terms of faster processing times.

V. CONCLUSION

FWL effects associated with the fixed-point implementation of $H_\infty$ controller structures of a teleoperated dual-wrist assembly control system have been studied. The relevant optimal and sparse FWL controller structures and their corresponding stability measures have been derived. The results presented illustrate the numerical and computational advantages of the finite-precision implementation of such critical teleoperated control system, while maintaining good closed-loop performance and improved stability characteristics.

Further ongoing work will explore the practical implementation issues of the resultant controller structures on fixed-point DSP platforms and to derive new stability measures which are less conservative. The integration of uncertainties and fragility issues of robust FWL controller realization are also being investigated to derive new compact controller structures with combined robust and enhanced numerical characteristics.

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